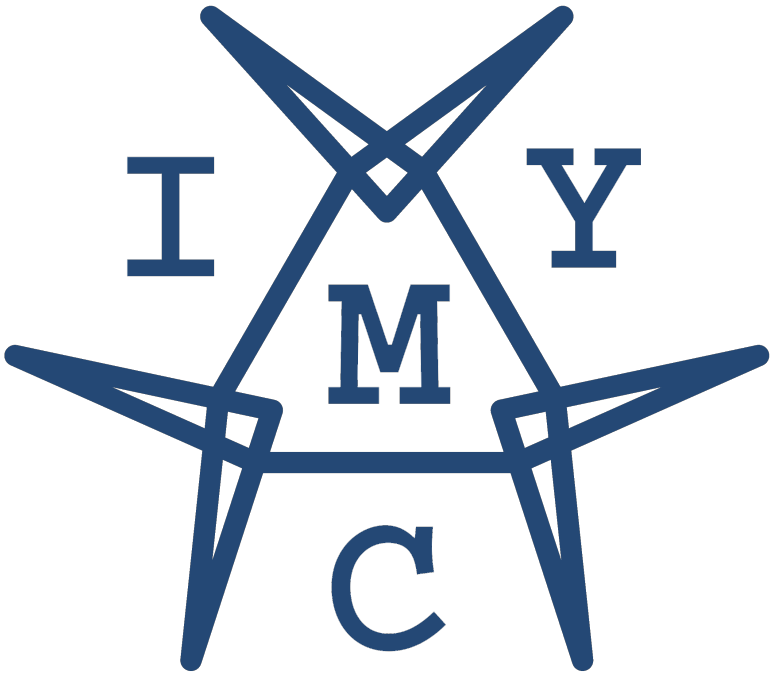


Final Round 2018

KEEP SECRET BEFORE AND AFTER FINAL EXAM!

Important: Read all the information on this page carefully!

General Information

- Please read all questions carefully!
- This exam consists of 30 multiple-choice questions.
- To every question, there are four possible answers: A, B, C and D.
- Only one of the four answer is correct!
- Every correct answer gives you one point.
- There are no negative points for wrong answers.
- You have strictly 60 minutes to solve as many problems as possible.
- If you cannot answer a question: Skip it! The final round consists of more questions than we expect you to answer.
- Write your answers on the *Your-Answers-page* only (see next page)!
- Following notation is used for the questions:
  - $x \in \mathbb{R}$ denotes a real number, $n \in \mathbb{N}$ denotes a positive integer.
  - f, g denote functions. (The domain and co-domain should follow from the context.)
  - The "roots" of a function f are those x such that $f(x) = 0$.
  - $\pi = 3.141...$ denotes the circle constant and $e = 2.718...$ Euler's number.
- **You are allowed to...**
  - use a pencil/pen for writing.
  - use extra blank papers for personal notes.
- **You are not allowed to...**
  - work more than 60 minutes on this exam.
  - use electronic devices (e.g. internet, calculators).
  - use any source of information (e.g. notes, books).
  - receive help from your supervisor or other students.
- **Cheating Policy:** In addition to the presence and supervision of your supervisor during the examination we have various additional methods to detect cheating: This includes methods to detect time violations as well as to detect the usage of tools (e.g. internet) for cheating. Cheating will result in immediate disqualification!

Good luck!

Your Answers

Your Name:

Please write your answers on this page!

No. 1	**No. 2**	**No. 3**	**No. 4**	**No. 5**	**No. 6**	**No. 7**	**No. 8**	**No. 9**	**No. 10**
No. 11	**No. 12**	**No. 13**	**No. 14**	**No. 15**	**No. 16**	**No. 17**	**No. 18**	**No. 19**	**No. 20**
No. 21	**No. 22**	**No. 23**	**No. 24**	**No. 25**	**No. 26**	**No. 27**	**No. 28**	**No. 29**	**No. 30**

Please write A, B, C or D into the boxes to give your answers.

Question 1 : What are the roots of the function $f(x) = x^2 + ax - 2a^2$ with $a \in \mathbb{R}$?

(A) $\{a, 2a\}$ (B) $\{-a, 2a\}$ (C) $\{a, -2a\}$ (D) $\{-a, -2a\}$

Question 2 : Find the point of intersection (x, y) of $f(x) = 2x - 3$ and $g(x) = -x^3$.

(A) $(1, 1)$ (B) $(-1, 1)$ (C) $(1, -1)$ (D) $(-1, -1)$

Question 3 : Solve this equation for $x \in \mathbb{R}$:

$$\frac{x-1}{2} - \frac{2x-3}{4} - \frac{3x-4}{5} = 3$$

(A) $x = \frac{39}{12}$ (B) $x = -\frac{39}{12}$ (C) $x = \frac{79}{24}$ (D) $x = -\frac{79}{24}$

Question 4 : What is the value of $\cos(7\pi/6)$?

(A) $1/2$ (B) $-1/2$ (C) $\sqrt{3}/2$ (D) $-\sqrt{3}/2$

Question 5 : Find the derivative $f'(x)$ of the function $f(x) = x^x$.

(A) $f'(x) = (\ln(x) + 1) \cdot x^x$ (B) $f'(x) = \ln(x + 1) \cdot x^x$

(C) $f'(x) = (\ln(x) + x) \cdot x^x$ (D) $f'(x) = \ln(x + x^x) \cdot x^x$

Question 6 : Find the numerical value of this expression:

$$\sin^2\left(\frac{\pi}{2}\right) \frac{\pi^{2-\cos(2\pi)}}{7 + (-1)^2} + \cos^2\left(\frac{\pi}{4} + \frac{\pi}{4}\right) \frac{\pi}{3^2 - 1}$$

(A) π (B) $\pi/2$ (C) $\pi/4$ (D) $\pi/8$

Question 7 : Find the value of this infinite sum: $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(A) $1/3$ (B) $1/4$ (C) 3 (D) 4

Question 8 : Which one is the set of prime numbers?

(A) $\{n \in \mathbb{N} \ : \ 1|n \ \wedge \ n|n\}$ (B) $\{n \in \mathbb{N} \ : \ 1|n \ \wedge \ n|n \ \wedge \ n > 1\}$

(C) $\{n \in \mathbb{N} \ : \ |\{t \in \mathbb{N} \ : \ t|n\}| = 2\}$ (D) $\{n \in \mathbb{N} \ : \ |\{t \in \mathbb{N} \ : \ t|n \ \wedge \ t > 1\}| = 2\}$

Question 9 : How can you express $\sin(x)$ with $\cos(x)$?

(A) $\sin(x) = \cos(\pi/2 + x)$ (B) $\sin(x) = \cos(\pi/2 - x)$

(C) $\sin(x) = \cos(\pi + x)$ (D) $\sin(x) = \cos(\pi - x)$

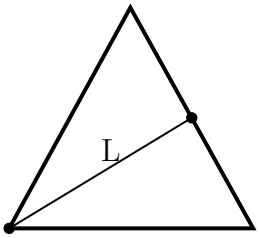
Question 10 : Which number divides $n^3 + 5n$ for all $n \in \mathbb{N}$?

(A) 4 (B) 5 (C) 6 (D) 7

Question 11 : Find the smallest value of the function $f(x) = x^2 + 2^{x^2}$ for $x \in \mathbb{R}$.

(A) 0 (B) 1/2 (C) 1 (D) 2

Question 12 : Below you find an equilateral triangle with all sides having a length of 1. A straight line intersects a corner and the center of the triangle. Find the length L of this line:



(A) $L = 1/2$ (B) $L = \sqrt{2}/2$ (C) $L = 3/2$ (D) $L = \sqrt{3}/2$

Question 13 : Solve this inequality for $x \in \mathbb{R}$:

$$\frac{5(9-x)}{3} + 1 < 16$$

(A) $x > 0$ (B) $x > 1$ (C) $x > 3$ (D) $x > 5$

Question 14 : Which equation is equal to $\cos(\alpha + \beta)$ with $\alpha, \beta \in \mathbb{R}$?

(A) $\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ (B) $\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

(C) $\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$ (D) $\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

Question 15 : What is the last digit of $11^5 \cdot 17^3 \cdot 33^3$?

(A) 1 (B) 2 (C) 3 (D) 5

Question 16 : Which number is also known as *Golden ratio*?

(A) $\frac{1+\sqrt{3}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$ (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{1-\sqrt{5}}{2}$

Question 17 : Find the value of x that is a solution to $\sqrt{2} = 4\sin(x)\cos(x)$.

(A) $x = \pi$ (B) $x = \pi/2$ (C) $x = \pi/4$ (D) $x = \pi/8$

Question 18 : Find the correct inequality for $a = \frac{1001}{1002}$, $b = \frac{1003}{1004}$ and $c = \frac{1002}{1003}$:

(A) $a < b < c$ (B) $a < c < b$ (C) $c < a < b$ (D) $c < b < a$

Question 19 : Find the correct inequality with $x = \pi^{\pi^2}$:

(A) $27^2 \leq x \leq 3^6$ (B) $3^6 \leq x \leq 3^9$ (C) $3^6 \leq x \leq 9^6$ (D) $27^2 \leq x \leq 3^9$

Question 20 : What are the roots $x \in \mathbb{R}$ of the function $f(x) = \frac{x^2-1}{x+2}$?

(A) $\{1, -1\}$ (B) $\{1, -\sqrt{2}\}$ (C) $\{-1, \sqrt{2}\}$ (D) $\{-1, -\sqrt{2}\}$

Question 21 : Find the solutions $x \in \mathbb{R}$ to this equation:

$$\pi \sin^2(x^3) - x^2 \cos^2(x^3) = x^2 \sin^2(x^3) - \pi \cos^2(x^3)$$

(A) $\{1, -1\}$ (B) $\{\sqrt{\pi}, -\sqrt{\pi}\}$ (C) $\{\pi, -\pi\}$ (D) $\{\pi^2, -\pi^2\}$

Question 22 : Find an expression for the product of the first k odd numbers:

$$P = 1 \cdot 3 \cdot 5 \cdot ... \cdot (2k-1)$$

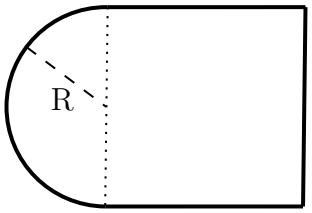
(A) $P = \frac{(2k)!}{2^k}$ (B) $P = \frac{(2k)!}{2^k k!}$ (C) $P = \frac{(2k+1)!}{2^k}$ (D) $P = \frac{(2k+1)!}{2^k k!}$

Question 23 : Find the sum of the first 100 numbers:

$$S = 1 + 2 + 3 + ... + 99 + 100$$

(A) 4950 (B) 5050 (C) 5150 (D) 5380

Question 24 : A semicircle with radius R is connected to a square (see below). Find the perimeter of this shape (length of bold line):



(A) $(3+\pi)R$ (B) $(3+2\pi)R$ (C) $(6+\pi)R$ (D) $(6+2\pi)R$

Question 25 : Which equation is true for any triangle with angles α, β, γ and sides a, b, c?

(A) $c^2 = a^2 + b^2 + 2ab\sin\gamma$ (B) $c^2 = a^2 + b^2 + 2ab\cos\gamma$

(C) $c^2 = a^2 + b^2 - 2ab\sin\gamma$ (D) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Question 26 : What is the value of this division: $\frac{20182018}{2018}$

(A) 101 (B) 1001 (C) 10001 (D) 100001

Question 27 : What is the value of $\frac{2x-3}{x-1}$ for $x \to \infty$?

(A) 0 (B) 1 (C) 2 (D) 3

Question 28 : How many last digits of 17! are zeros?

(A) 2 (B) 3 (C) 4 (D) 5

Question 29 : What is the value of $S = \sum_{n=1}^{\infty} \frac{1}{n^2}$?

(A) $1/2$ (B) $\pi/3$ (C) $\pi^2/6$ (D) $\pi^3/9$

Question 30 : How likely have at least two of 23 students birthday on the same day?

(A) $\approx 10\%$ (B) $\approx 30\%$ (C) $\approx 50\%$ (D) $\approx 70\%$