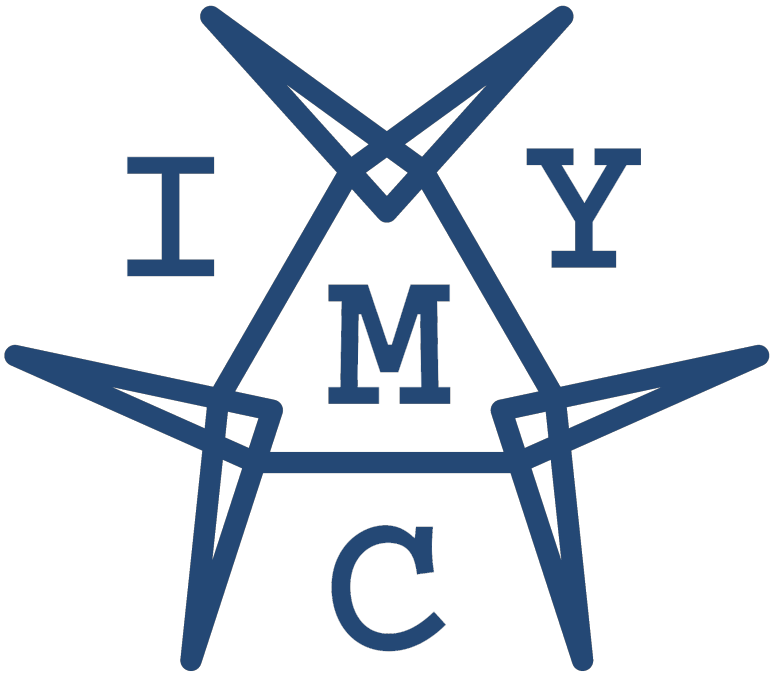


Final Round 2019

KEEP SECRET BEFORE AND AFTER FINAL EXAM!

Important: Read all the information on this page carefully!

General Information

- Please read all questions carefully!
- This exam consists of 40 multiple-choice questions.
- To every question, there are four possible answers: A, B, C and D.
- Only one of the four answer is correct!
- Every correct answer gives you one point.
- There are no negative points for wrong answers.
- You have strictly 60 minutes to solve as many problems as possible.
- If you cannot answer a question: Skip it! The final round consists of more questions than we expect you to answer. We do not expect you to answer all questions.
- Write your answers on the *Your-Answers-page* only (see next page)!
- Following notation is used for the questions:
  - $x \in \mathbb{R}$ denotes a real number, $n \in \mathbb{N}$ denotes a positive integer.
  - f, g denote functions. The domain and co-domain should follow from the context.
  - The "roots" of a function f are those x such that $f(x) = 0$.
  - $\pi = 3.141...$ denotes the circle constant and $e = 2.718...$ Euler's number.
- **You are allowed to...**
  - use a pencil/pen for writing.
  - use extra blank papers for personal notes.
- **You are not allowed to...**
  - work more than 60 minutes on this exam.
  - use electronic devices (e.g. internet, calculators).
  - use any source of information (e.g. notes, books).
  - receive help from your supervisor or other students.
  - keep the final exam (problems and answers) after the exam.
- **Cheating Policy:** In addition to the presence and supervision of your supervisor during the examination we have various additional methods to detect cheating: This includes methods to detect time violations as well as to detect the usage of tools (e.g. internet) for cheating. Cheating will result in immediate disqualification!

Good luck!

Your Answers

Your Name:

Cross the box for A,B,C or D to give your answer to a question (like this: ☒).
To correct yourself, please draw out the box of your *new* answer (like this: ■).

	A	**B**	**C**	**D**
Question 1	☐	☐	☐	☐
Question 2	☐	☐	☐	☐
Question 3	☐	☐	☐	☐
Question 4	☐	☐	☐	☐
Question 5	☐	☐	☐	☐
Question 6	☐	☐	☐	☐
Question 7	☐	☐	☐	☐
Question 8	☐	☐	☐	☐
Question 9	☐	☐	☐	☐
Question 10	☐	☐	☐	☐
Question 11	☐	☐	☐	☐
Question 12	☐	☐	☐	☐
Question 13	☐	☐	☐	☐
Question 14	☐	☐	☐	☐
Question 15	☐	☐	☐	☐
Question 16	☐	☐	☐	☐
Question 17	☐	☐	☐	☐
Question 18	☐	☐	☐	☐
Question 19	☐	☐	☐	☐
Question 20	☐	☐	☐	☐
Question 21	☐	☐	☐	☐
Question 22	☐	☐	☐	☐
Question 23	☐	☐	☐	☐
Question 24	☐	☐	☐	☐
Question 25	☐	☐	☐	☐
Question 26	☐	☐	☐	☐
Question 27	☐	☐	☐	☐
Question 28	☐	☐	☐	☐
Question 29	☐	☐	☐	☐
Question 30	☐	☐	☐	☐
Question 31	☐	☐	☐	☐
Question 32	☐	☐	☐	☐
Question 33	☐	☐	☐	☐
Question 34	☐	☐	☐	☐
Question 35	☐	☐	☐	☐
Question 36	☐	☐	☐	☐
Question 37	☐	☐	☐	☐
Question 38	☐	☐	☐	☐
Question 39	☐	☐	☐	☐
Question 40	☐	☐	☐	☐

Question 1 : What are the roots of the function $f(x) = \frac{x^2-4x+3}{2^x-4}$?

(A) $\{1, 3\}$ (B) $\{1, 4\}$ (C) $\{-1, 3\}$ (D) $\{-1, 4\}$

Question 2 : Let $a = 2$, $b = 3$ and $c = a^2$. What is the numerical value of $a^c + b^b + c^a$?

(A) 58 (B) 59 (C) 60 (D) 61

Question 3 : How does this sequence of numbers continue?: $7,\ 26,\ 63,\ 124, ...$

(A) 205 (B) 215 (C) 225 (D) 235

Question 4 : What is the value of $\sin(150°) + \cos(4\pi/3)$?

(A) $-1/2$ (B) 0 (C) $1/2$ (D) 1

Question 5 : Find the result of this division: $\frac{111111}{11}$

(A) 10001 (B) 10101 (C) 10110 (D) 11111

Question 6 : Determine the value of z with these three equations:

$$x = 3^2 + 2^3\ , \quad y = x^2 + 1\ , \quad z = (y + x)^2$$

(A) $z = 94249$ (B) $z = 94349$ (C) $z = 94449$ (D) $z = 94549$

Question 7 : The volume of a sphere with radius r is equal to ...

(A) πr^3 (B) $4\pi r^3$ (C) $\pi r^3/3$ (D) $4\pi r^3/3$

Question 8 : Find the smallest k such that $2^k > 1000000$:

(A) $k = 14$ (B) $k = 16$ (C) $k = 18$ (D) $k = 20$

Question 9 : Find the derivative $f'(x)$ of the function $f(x) = \sin(x) \cdot e^{x^2}$?

(A) $f'(x) = \cos(x)e^{x^2}(1 + 2x \sin x)$ (B) $f'(x) = \sin(x)e^{x^2}(1 + 2x \sin x)$

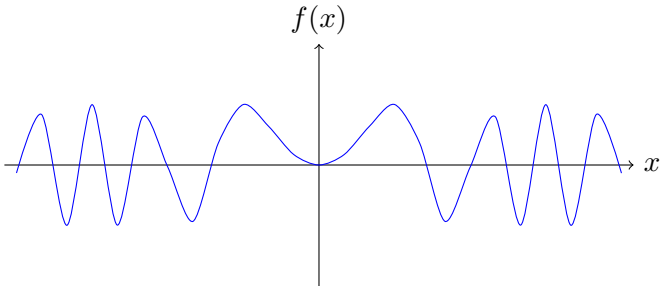
(C) $f'(x) = \cos(x)e^{x^2}(1 + 2x \tan x)$ (D) $f'(x) = \sin(x)e^{x^2}(1 + 2x \tan x)$

Question 10 : Calculate the numerical value:

$$\left(\frac{3}{2}\right)^4 - \frac{9 \cdot (8-1) + \log_{11} 121}{17 + (-1)^{77}} + \left(6^2 - \frac{210}{35}\right)^2$$

(A) 899 (B) 900 (C) 901 (D) 902

Question 11 : Find the function $f(x)$ with this graph:



(A) $f(x) = \sin(x^2)$ (B) $f(x) = \sin^2(x)$ (C) $f(x) = \sin^2(x^2)$ (D) $f(x) = \sin(1/x)$

Question 12 : Which number divides $5 + 4^n$ for all positive integers n?

(A) 3 (B) 4 (C) 5 (D) 9

Question 13 : Determine the value of this alternating sum:

$$\sum_{n=1}^{1550} (-1)^n \cdot n$$

(A) 225 (B) 775 (C) 1549 (D) 1550

Question 14 : A *transcendental number* is a number that is ...

(A) not the limit of a rational sequence. (B) not rational and also not irrational.

(C) not a root of an integer polynomial. (D) not the solution of a continued fraction.

Question 15 : What are the roots of the function $f(x) = \pi^3 - (\pi + \pi^2 + \pi^3)x + (1 + \pi + \pi^2)x^2 - x^3$?

(A) $\{1, \pi, \pi^2\}$ (B) $\{\pi, \pi^2, \pi^3\}$ (C) $\{-1, \pi, \pi^2\}$ (D) $\{-\pi, \pi^2, \pi^3\}$

Question 16 : Find the value of this expression: $\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)$

(A) $\frac{1}{2}\sqrt{2} + \sqrt{3}$ (B) $\sqrt{2} + \frac{1}{2}\sqrt{3}$ (C) $\frac{3}{2}\sqrt{2} + \frac{1}{2}\sqrt{3}$ (D) $\frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{3}$

Question 17 : For which n is $p_n = n^2 - n + 41$ not a prime number?

(A) 41 (B) 13 (C) 27 (D) 60

Question 18 : What is the mathematical form of the *Mersenne primes* M_p?

(A) $M_p = 2^p - 1$ (B) $M_p = 2^p + 1$

(C) $M_p = 2^{p-1} - 1$ (D) $M_p = 2^{p-1} + 1$

Question 19 : Find the correct $f(x)$ such that this identity holds true:

$$\frac{d}{dx}\arctan(x) = \frac{1}{f(x)}$$

(A) $f(x) = 1 + x^2$ (B) $f(x) = 1 - x^2$ (C) $f(x) = \sqrt{1 + x^2}$ (D) $f(x) = \sqrt{1 - x^2}$

Question 20 : Solve this equation for x: $2\log_2(x - 1) = 3 + \sin(\pi/2)$

(A) 2 (B) 3 (C) 4 (D) 5

Question 21 : The binary representation of the decimal number 127 is ...

(A) 1111100 (B) 1111101 (C) 1111110 (D) 1111111

Question 22 : Let $\sigma(n)$ be the sum-of-divisors function. What is $\sigma(101)$?

(A) 101 (B) 102 (C) 103 (D) 104

Question 23 : Find the condition that makes this identity true for α, β, γ:

$$\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \cdot \tan\beta \cdot \tan\gamma$$

(A) $\alpha + \beta + \gamma = 45°$ (B) $\alpha + \beta + \gamma = 90°$

(C) $\alpha + \beta + \gamma = 180°$ (D) $\alpha + \beta + \gamma = 360°$

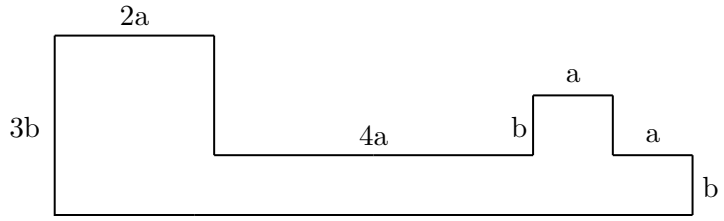
Question 24 : What is the 10th derivative $f^{(10)}(x)$ of the function $f(x) = 1/x$?

(A) $10!/x^{10}$ (B) $-10!/x^{10}$ (C) $-10!/x^{11}$ (D) $10!/x^{11}$

Question 25 : Calculate the numerical value of this sum: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

(A) $\frac{40}{20}$ (B) $\frac{43}{20}$ (C) $\frac{47}{20}$ (D) $\frac{49}{20}$

Question 26 : Find the perimeter of the shape below (i.e., length of the black line):



(A) $8a + 6b$ (B) $8a + 8b$ (C) $16a + 6b$ (D) $16a + 8b$

Question 27 : The famous question *"Can every even integer greater than 2 be expressed as the sum of two primes?"* is also known as ...

(A) Riemann hypothesis (B) Poincaré conjecture

(C) Prime Number Theorem (D) Goldbach's conjecture

Question 28 : How many digits are in the number 25! (i.e., 25 factorial)?

(A) 26 (B) 27 (C) 28 (D) 29

Question 29 : Find a function $f(x)$ such that the derivative $f'(x) = 3^x \cdot x^3 \cdot (\ln 3^x + 3 + 1)$?

(A) $f(x) = 3^x \cdot x^3$ (B) $f(x) = 2 \cdot 3^x \cdot x^3$ (C) $f(x) = 3 \cdot 3^x \cdot x^3$ (D) $f(x) = x \cdot 3^x \cdot x^3$

Question 30 : Which one is the best numerical approximation of π?

(A) 22/7 (B) $\sqrt{4e-1}$ (C) $\sqrt[1/3]{31}$ (D) 512/163

Question 31 : Find the correct equation which calculates this sum: $S = 1^2 + 2^2 + ... + n^2$

(A) $S = \frac{n(n+1)(2n+1)}{3}$ (B) $S = \frac{n(n+1)(2n-1)}{3}$ (C) $S = \frac{n(n+1)(2n+1)}{6}$ (D) $S = \frac{n(n+1)(2n-1)}{6}$

Question 32 : Calculate the value of this fraction: $\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}}$

(A) 5/8 (B) 4/6 (C) 6/5 (D) 4/8

Question 33 : *Fermat's Last Theorem* asks for solutions to ...

(A) $a^n + b^n = c^n$ (B) $e^{i\pi} + 1 = 0$ (C) $a^p \equiv a \pmod{p}$ (D) $1 + \frac{1}{2^n} + \frac{1}{3^n} + ...$

Question 34 : The value of $\cos(x)$ is equal to ...

(A) $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{(2n)!}$ (B) $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{(2n+1)!}$ (C) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ (D) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$

Question 35 : Let $\sigma(n)$ be the sum-of-divisors function. Which conditions holds true for $6, 28, 496, 8128$?

(A) $\sigma(n) = n$ (B) $\sigma(n) = 2n$ (C) $\sigma(n) > n$ (D) $\sigma(n) < 2n$

Question 36 : Calculate the value of this sum: $\frac{1+2}{3+4} + \frac{3+4}{1+2} + \frac{1+4}{2+3} + \frac{2+3}{1+4}$

(A) 100/19 (B) 100/21 (C) 100/23 (D) 100/27

Question 37 : Find the numerical value of $\frac{1}{2}\log_2(4e^2) - \frac{1}{\ln(2)}$.

(A) 0 (B) 1 (C) ln(2) (D) 1/2

Question 38 : Which one of the following sets is equal to $[\frac{1}{2}, 1) \cup [\frac{1}{3}, 1) \cup [\frac{1}{4}, 1) \cup ...$?

(A) $(0, 1)$ (B) $[0, 1)$ (C) $(0, 1]$ (D) $[0, 1]$

Question 39 : What is the probability to throw a dice six times without getting a six?

(A) $\approx 16\%$ (B) $\approx 33\%$ (C) $\approx 66\%$ (D) $\approx 83\%$

Question 40 : You have given a triangle with two sides of equal length. Determine the length of the third side given the circumference U to maximize the area of the triangle.

(A) $U/2$ (B) $U/3$ (C) $U/4$ (D) $U/5$