International Youth Math Challenge

Qualification Round 2020

Problem A

Find the roots of the function $f(x) = (2^x - 1) \cdot (x^2 + 2x - 3)$ with $x \in \mathbb{R}$.

Problem B

Show that $3 \cdot 4^n + 51$ is divisible by 3 and 9 for all positive integers n.

Problem C

Determine the numerical value of the following expression without the use of a calculator:

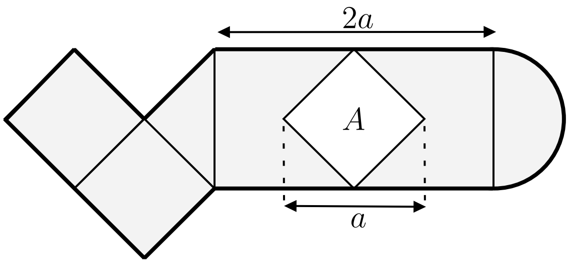
$$\sqrt{\sin\left(\frac{\pi + \log_2\left(\sqrt{2^\pi \cdot 2^\pi}\right)}{2^5 - 2^4 - 2^3}\right) \cdot \sqrt[3]{\frac{2^{4-1}}{2^{3/2}}} + \log_2\left(\log_3(9^{15}) + \pi^{1+(-1)^{17}} + 1\right) + \frac{(-1)^5 + (-1)^{27}}{(-1)^{766}}}$$

Problem D

Prove that the inequality $|\cos(x)| \geq 1 - \sin^2(x)$ holds true for all $x \in \mathbb{R}$.

Problem E

The white square in the drawing below is located in the centre of the grey rectangle and has a surface area of A. The width of the rectangle is twice the width a of the square. What is the surface of the grey area (without the white square)?



Submission Information
To qualify for the next round, you have to solve at least three/four (under/over 18 years) problems correctly.
Show your steps! Make sure to submit your solution until *Sunday 11. October 2020 23:59 UTC+0* online!
Further information and the submission form is available on the competition website: www.iymc.info