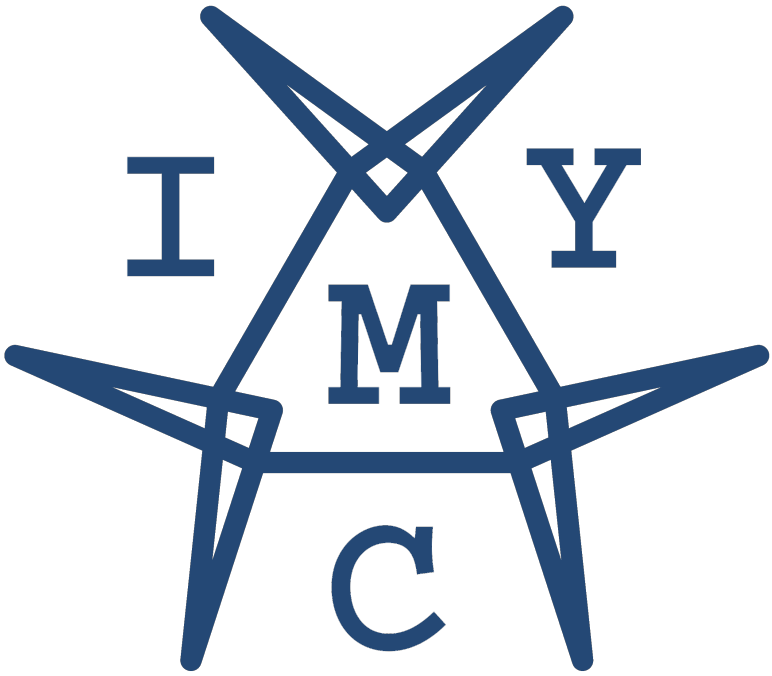


Final Round 2023

The final round exam was given in the form of an online exam. Each participant was given a subset of 20 questions in random order. This paper version is only available for training purposes.

Question 1 : What are the roots of this function?:

$$f(x) = \left(\pi^x - \frac{1}{\pi^2}\right) \cdot \left(\frac{1}{\pi^2} - \pi^{2x}\right)$$

(A) $\{-1, -2\}$ (B) $\{-1, 2\}$ (C) $\{1, -2\}$ (D) $\{1, 2\}$

Question 2 : Which of these numbers is divisible by 3 for all positive integers n?

(A) $2^n + (-1)^{n+1}$ (B) $2^n + (-1)^n$ (C) $2^n - (-1)^{n+1}$ (D) $2^{n+1} - (-1)^n$

Question 3 : Let $\sigma(n)$ be the sum of all positive divisors of the integer n. What is the value of $\sigma(17^2)$?

(A) 237 (B) 290 (C) 307 (D) 337

Question 4 : What is the value of this sum?:

$$\sum_{n=0}^{100} \left[(-1)^n - (-1)^{n+1}\right]$$

(A) 2 (B) 1 (C) -1 (D) -2

Question 5 : What is the ratio between the surface area and the circumference of an equilateral triangle with side length a?

(A) $\sqrt{3} \cdot a$ (B) $\frac{\sqrt{3}}{4} \cdot a$ (C) $\frac{1}{\sqrt{3}} \cdot a$ (D) $\frac{1}{4\sqrt{3}} \cdot a$

Question 6 : Let $f(x) = 4 + 4x$ and $g(x) = 1 - 2x$. Find the smallest $a \in \mathbb{R}$ such that $f(x) > g(x)$ for all x with $x > a$.

(A) $a = -1$ (B) $a = -1/2$ (C) $a = 1/2$ (D) $a = 1$

Question 7 : Find the derivative $f'(x)$ of this function:

$$f(x) = x^x$$

(A) $f'(x) = (\log(x) + 1/x) \cdot x^{x-1}$ (B) $f'(x) = (\log(x) + 1) \cdot x^{x-1}$

(C) $f'(x) = (\log(x) + 1/x) \cdot x^x$ (D) $f'(x) = (\log(x) + 1) \cdot x^x$

Question 8 : What is the value of this infinite sum?:

$$\sum_{n=0}^{\infty} \frac{2^n}{(1^n+2)^n}$$

(A) 1/2 (B) 1/3 (C) 2 (D) 3

Question 9 : How does this sequence of numbers continue?:

$$1,\ 2,\ 6,\ 12,\ 36,\ 72, ...$$

(A) 144 (B) 216 (C) 244 (D) 286

Question 10 : The Riemann zeta function is defined by $\zeta(s) = ...$

(A) $\sum_{n=0}^{\infty} 1/n^s$ (B) $\sum_{n=1}^{\infty} 1/n^s$ (C) $\sum_{n=0}^{\infty} 2^n/n^s$ (D) $\sum_{n=1}^{\infty} 2^n/n^s$

Question 11 : Which $f(x)$ makes this equation true?:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{\text{p prime}} \frac{1}{f(x)}$$

(A) $f(x) = 1 - p^{-s}$ (B) $f(x) = 1 + p^{-s}$ (C) $f(x) = 1 - p^s$ (D) $f(x) = 1 + p^s$

Question 12 : Let $\omega(n)$ be the number of distinct prime factors of n. What is the value of $\omega(60)$?

(A) 2 (B) 3 (C) 4 (D) 5

Question 13 : Let $\omega(n)$ be the number of distinct prime factors of n. Under which condition is $\omega(mn) = \omega(m) + \omega(n)$?

(A) $m|n$ (B) $n|m$ (C) $\text{lcd}(m, n) = 1$ (D) $\gcd(m, n) = 1$

Question 14 : What is the value of $\zeta(2)$?:

$$\zeta(2) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + ...$$

(A) $1/2$ (B) 2 (C) $\pi^2/3$ (D) $\pi^2/(2 \cdot 3)$

Question 15 : Find the numerical value of this expression:

$$\sin\left(\frac{\pi}{6} \cdot \log_2(3+5)\right)$$

(A) 0 (B) -1 (C) 1 (D) $\sqrt{2}$

Question 16 : What are the roots of this function?:

$$f(x) = x^4 + 2x^2 - 3$$

(A) $\{-3, -1\}$ (B) $\{-2, -1\}$ (C) $\{-1, 1\}$ (D) $\{-1, 2\}$

Question 17 : You are given the following functions:

$$f(x) = Ax^3 + 3$$

$$g(x) = Bx^2 + 2$$

$$h(x) = Cx - 1$$

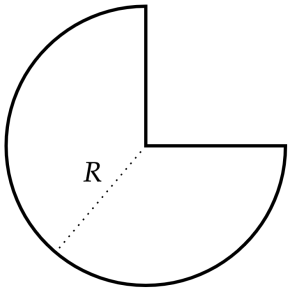
They all intersect the point $(1, 1)$. What is the value of the product $A \cdot B \cdot C$?

(A) 0 (B) 1 (C) -2 (D) 4

Question 18 : Let $\varphi(x) = x^2 + 1$. What is the numerical value of $\varphi(\varphi(\varphi(2)))$?

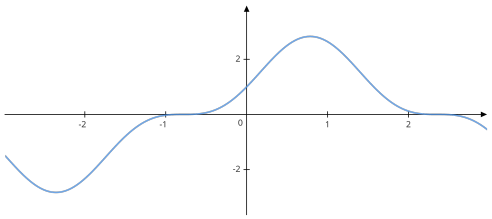
(A) 26 (B) 432 (C) 677 (D) 1458

Question 19 : You are given a circle with a radius R of which $1/4$ is missing. What is the circumference of this figure (see drawing)?:



(A) $\left(\frac{3}{2}\pi + 2\right) \cdot R$ (B) $\left(\frac{3}{4}\pi + 2\right) \cdot R$ (C) $(2\pi + 2) \cdot R$ (D) $(3\pi + 2) \cdot R$

Question 20 : Given the following graph, find the function $f(x)$:



(A) $f(x) = (\sin(x) - \cos(x))^3$ (B) $f(x) = (\sin(x) - \cos(x))^2$

(C) $f(x) = (\sin(x) + \cos(x))^3$ (D) $f(x) = (\sin(x) + \cos(x))^2$

Question 21 : The expression $(2 + 2^x)^2$ is equal to ...

(A) $4 + (4 + 2^x) \cdot 2^{2x}$ (B) $4 + (4 + 2^x) \cdot 2^x$

(C) $4 + (4 + 2^{2x}) \cdot 2^{2x}$ (D) $4 + (4 + 2^{2x}) \cdot 2^x$

Question 22 : Let $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$ and $\gamma = \frac{1}{4}$. What is the numerical value of this fraction?:

$$\frac{\alpha + \beta + \gamma}{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}$$

(A) 1/24 (B) 1/108 (C) 13/24 (D) 13/108

Question 23 : Which x solves this equation?:

$$\frac{1}{1 + \frac{1}{x}} = \frac{2 \cdot 3}{2^3 - 1}$$

(A) 2 (B) 4 (C) 6 (D) 8

Question 24 : Solve this inequality for $x \in \mathbb{R}$ and $x > 0$:

$$\frac{1}{x} + \frac{2}{3} > \frac{1}{2x} + 3$$

(A) $x < 3/13$ (B) $x < 6/13$ (C) $x < 3/14$ (D) $x < 6/14$

Question 25 : Which inequality is true for this number?:

$$z = \sqrt{1+\sqrt{2}} + \sqrt{3+\sqrt{4}}$$

(A) $1 \leq z \leq 2$ (B) $2 \leq z \leq 3$ (C) $3 \leq z \leq 4$ (D) $4 \leq z \leq 5$

Question 26 : You are given five multiple-choice questions. Each question has three incorrect choices and one correct choice. How likely do you answer none of them correctly if you guess randomly?

(A) $\approx 15\%$ (B) $\approx 25\%$ (C) $\approx 35\%$ (D) $\approx 45\%$