International Youth Math Challenge

Qualification Round 2023

Problem A

What are the roots of the function $f(x) = \left(\pi^x - \frac{1}{\pi}\right) \cdot x^\pi \cdot \left(\frac{1}{\pi^2} - \pi^x\right)$ with $x \in \mathbb{R}$?

Problem B

Show that $2^n - (-1)^n$ is divisible by 3 for all positive integers n.

Problem C

Determine the numerical value of the following expression without the use of a calculator:

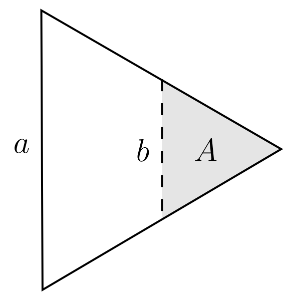
$$\log_3\left(\frac{\sin(2\pi)}{\tan(\pi/3)} + \sum_{n=0}^{10}\left(\frac{1+\sqrt{1+\sqrt{1}}}{1+(1^{-1}+1^{1-1})\cdot\cos(\pi/4)} - \frac{2^4-2^3}{(1+\sqrt{1})^2}\right)^n\right)$$

Problem D

Let $\sigma(n)$ be the sum of all positive divisors of the integer n and let p be any prime number. Prove that $\sigma(p^m) = 1 + p \cdot \sigma(p^{m-1})$ for all positive integers m.

Problem E

The drawing below shows an equilateral triangle with side length a. A vertical line of length b intersects the triangle (dotted line; $a \parallel b$). Find the area A of the enclosed triangle (grey area).



Submission Information

To qualify for the next round, you have to solve at least three/four (under/over 18 years) problems correctly. Show your steps! Make sure to submit your solution until *Sunday 17. September 2023 23:59 UTC+0* online! Further information and the submission form is available on the competition website: www.iymc.info